Classical and quantum trampoline for ultra-cold atoms, in Quantum phenomena in gravitational field

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Classical and quantum trampoline for ultra-cold atoms

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Cold atomic gases have proven to be a useful resource for precision measurements of the atom properties or of the external forces acting on them. For example, atom interferometers permit the measurement of the local gravity constant *g* with a relative accuracy of the order of 10^{-8} at 1s [1]. A long observation time is demanded in order to obtain the best accuracy. This time is limited not only by the expansion of the sample and thus its temperature (100 nK corresponds to an expansion velocity of ~3 m/s for Rubidium 87 and can be achieved by a combination of laser cooling and evaporative cooling techniques), but also by the size of the vacuum chamber in which the atoms are in free fall. In this paper, we describe two schemes producing atomic mirrors[2] in order to bounce the atoms upward in a controllable way and thus keep them in a small volume for a long time. These two schemes are hereafter called classical[3,4] and quantum trampolines[5].

The mirrors for both schemes are based on atom diffraction by a periodic optical potential[6], *i.e.* a vertical light standing wave of period π/k_L, where k_L is the laser wavevector (in our case $2\pi/k_L$=780 nm). An atom in the momentum state $|-k_L>$ can be reflected to the state $|+k_L>$ by a standing wave laser pulse. This is a process called Bragg diffration. After a time $T_0 = 2\ \hbar k_L/mg \approx 1.2$ ms for ^{87}Rb, where $\hbar$ is the reduced Planck constant and *m* the atomic mass[7], the state $|+k_L>$ evolves back into the state $|-k_L>$ because of the downwards acceleration of gravity *g*. Repeating the standing-wave laser pulse with a period T_0 thus allows to suspend the atoms at an almost constant altitude. It realizes a trampoline for ultra-cold atoms. This setup allows for the measurement of *g*[3,4] as the trampoline only suspends atoms for the precise value of the period T_0.

We first analyse more precisely the diffraction process. The Hamiltonian *H* of an atom in the presence of the standing wave and of gravity reads: $H=p^2/2m+mgz+V\sin^2(2k_Lz)$. In the absence of the standing wave (V=0), a momentum state $|k>$ evolves with time t in $|k-gt/\hbar>$. The interaction between the atoms and the optical potential leads to vertical momentum changes quantized in units of $2\hbar k_L$. Starting with a momentum state $|k>$ at time t=0, only the states $|k-gt/\hbar+2nk_L>$, where *n* is an integer, will be populated over time. In this basis, the Hamiltonian reduces to a simple matrix with the energies $\hbar^2/2m\ (k-mgt/\hbar+2nk_L)^2$ along the diagonal and $V/4$ as the coupling between the states differing by $2k_L$. The system dynamics can be simulated numerically.

To use the standing wave as a mirror, the idea is to use the resonant coupling between the states $|-k_L>$ and $|+k_L>$, which have the same energy. In this case a perfect mirror should be realizable. However, as the momentum states are constantly changing due to gravity, the resonant condition is only transiently met, reducing the possible duration of the pulse. In order to realize a good mirror (avoiding higher order diffraction), it is also favourable to use a smoothly varying intensity of the standing wave rather than square-shaped pulsed[8,9]. In our case, we use a laser intensity varying as $\cos^2(\pi\ t/\tau)$ between t=-τ/2 and τ/2, where τ is the total duration of the pulse and where the resonant condition is met at t=0. For each value of τ, the absolute value of the intensity is adjusted in order to obtain the maximum amount of transfer from the state $|-k_L>$ to $|+k_L>$. Theoretically, we find that τ=170 μs gives an optimum transfer efficiency (larger than 0.999). Figure 1 shows the different state occupation probabilities as a function of time during the pulse.

In the experiment, we use τ=180 μs and are able to bounce a cloud of atoms 25 times corresponding to a total time of 30 ms (see [10] for details on the cold atom sample preparation). By varying the period *T* between the laser pulses, we observe a resonance in the number of atoms kept on the trampoline[3,4] (see figure 2). This effect can be simply understood. If the period *T* does not match T_0, the atoms have a mean residual acceleration and later laser pulses are applied on atoms with a momentum different from $-\hbar k_L$. The reflection probability is then reduced. From our experimental data, we can estimate T_0 and thus also give an estimate of the value of gravity $g=9.81(1)$. This method does not rely on the phase between the different pulses as it is not an interferometer. We call it a classical trampoline[11]. The suspended atoms can then by used to build a two-path interferometer has demonstrated in [4].

Interestingly, our setup can also efficiently suspend atoms against gravity using shorter and square-shaped pulses[5]. This may seem surprising as in this case, the quality of an individual atomic mirror is poor. For example, for a 35 μs pulse acting on $|-k_L\rangle$, 93% of the atoms are transferred to $|+k_L\rangle$, but also 3% to $|-3k_l\rangle$ and $|+3k_l\rangle$ and a fraction below 1% remains in $|-k_L\rangle$. Experimentally, in this case, we observe that (unlike in the smoothed pulse case) the fraction of suspended atoms shows fringes as a function of T (see figure 3). This behavior is characteristic of an interferometer. These interferences actually arise from the recombination of the various trajectories populated due to the splitting induced by the imperfect mirror pulses, as detailled in [5] and similarly to the theoretical proposition[12]. This method thus permits to simultaneously suspend the atom against gravity and create an interferometer. Actually, one can use the absolute fringe position for a measurement of gravity, as in standard atom gravimeters. In order to do that, one has to be able to predict the phase acquired during the laser pulses (using the diffraction model presented previously). For example, the phase of the interferometer varies with the laser pulse duration τ (see Fig. 3b). Here we find g= 9.815(4) in agreement with the expected value in Palaiseau[13].

In conclusion, we have shown methods to suspend atoms against gravity using a standing-wave as an atomic mirror. Using smoothed standing-wave laser pulses, very efficient reflection is expected and the atoms are suspended as on a trampoline. Alternatively, short square-shaped laser pulses lead to numerous possible atom trajectories, which then interfere. The atoms may then efficiently bounce due to quantum interferences hindering the losses. In this case, our setup is a quantum trampoline[5], a multiple-wave interferometer[14,15,16] permitting the measurement of gravity. Our work opens perspectives for new types of compact interferometers, in which atoms do not fall over an extended distance.

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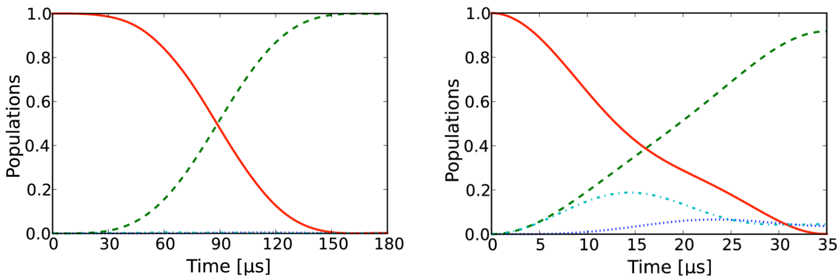

Figure 1: Evolution of the occupation probability of the different momentum states as a function of time for a smooth standing-wave light pulse of duration 180 μs (left) and for a square-shaped pulse of duration 35 μs (right). Solid red line: $|-k_L>$, dashed green line: $|+k_L>$, dot-dashed light blue line: $|-3k_L>$, dotted blue line: $|+3k_L>$.

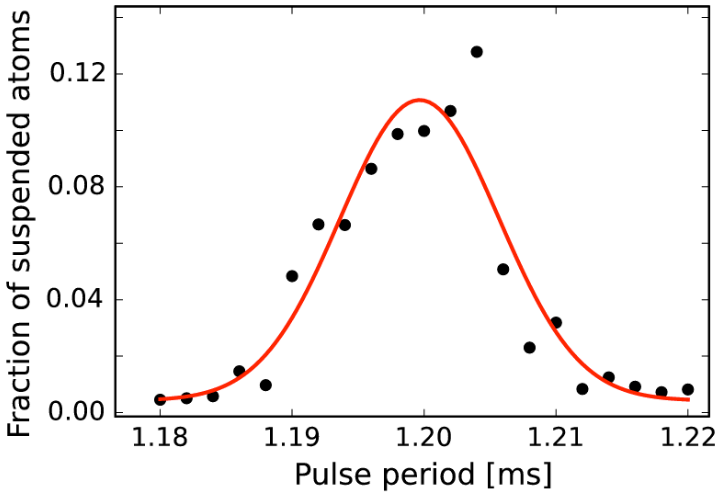

Figure 2: Fraction of suspended atoms after 25 Bragg reflections using smooth light pulses as a function of the period *T*. Solid line: Gaussian fit to the data centred at *T*=1.1996(10) ms, corresponding to *g*=9.81(1).

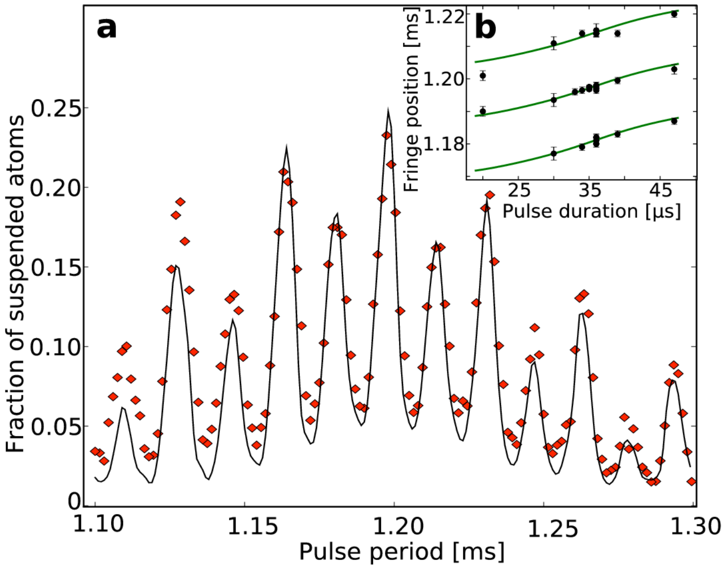

Figure 3: Fraction of suspended atoms after 10 square-shaped light pulses as a function of the period *T*. The overall envelope is due to the velocity selectivity[3] of the pulses as in the classical trampoline while the modulation is due to quantum interference. The solid line corresponds to a theoretical model[5]. **b**, position of three consecutive fringe maxima around the highest maximum, as a function of the pulse duration, showing the influence of the phase shift imprinted by the diffraction pulses : Dots are experimental points, with error bars reflecting the experimental uncertainties. Solid lines come from a theoretical model using g=9.815 m.s^{-2}.